INDIAN STATISTICAL INSTITUTE

Final Examination (Academic Year 2020-21, First Semester)
B. Math III Year
Introduction to Stochastic Processes

Teacher: Parthanil Roy

Date: 23/12/2020   **Maximum Marks: 40**   **Duration: 10:00 am - 1:30 pm**

Note:
- **Please write your name on top of your answer paper and submit it as one PDF file.**
- **You may use any theorem proved or stated in the class but do not forget to quote the appropriate result.**
- **If needed, please feel free to contact the teacher at parthanil.roy@gmail.com.**

1. Suppose a typewriter only has the keys A - Z. A monkey decides to type using that typewriter at random. What is the expected number of keys the monkey has to press to type the word **MARKOV** for the first time? Please justify your answer.   [8]

2. Show that positive recurrence is a class property.   [8]

3. Suppose two types of passengers arrive in a boat ferry point with unlimited supply of ferry boats. The first type of passengers will wait patiently till *four passengers* (= capacity of a ferry boat) arrive and the second type will simply reserve the boat and go away immediately. Assume that these two types of passengers arrive independently according to Poisson processes with rates 10 and 5 per hour, respectively. Priorities are given to second type of passengers even if a few passengers of the first type are waiting in the queue. However, while each of the first type of passengers pay Rs. 10 for the trip, the second type of passengers are charged Rs. 60. Assume also that no time is lost in passengers getting into the ferry boat, the driver takes the money from the passenger(s) and departs immediately without wasting any time. Given that exactly 6 passengers arrive during 9 : 00 am - 9 : 30 am, compute the expected total earning of ferry boat drivers in that time-span.   [8]

4. Suppose $\{X_n\}_{n\geq 0}$ is a Markov chain with state space S. For any subset $A \subseteq S$, define $cl(A)$ (the closure of A) to be the smallest closed subset of S containing A. Show that
$$cl(\{j\}) = \{k \in S : j \to k\}$$
for any state $j \in S$. If j is recurrent, then show that $cl(\{j\})$ is the communication class containing j.   [4 + 4]

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5. Suppose $\{X_n\}_{n\geq0}$ is a Markov chain with a stationary distribution π. Show that
$$\{(X_n, X_{n+1})\}_{n\geq0}$$
is also a Markov chain. Find out its stationary distribution. $[4+4]$

Wish you all the best